1

* Complex Analysis *

→ Numbers systems ↔

1) Natural Numbers (1, 2, 3, ...)

2) integr Numbers (-3, -2, -1, 0, 1, 2, 3)

3) Rational Numbers ($a/b : a \neq b$)

4) Real Numbers R

5) Complex → $x + iy$ $i = \sqrt{-1}$ $\mathbb{C}$

$z = x + iy$ $x = R_e(z)$ $y = I_m(z)$

= Properties of Complex numbers =

I) $i = \sqrt{-1}$ II) $i^2 = -1 \to i^2 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = \sqrt{-1 \times -1} = 1$

III) addtion $z_1 = x_1 + iy_1$ $z_2 = x_2 + iy_2$

$z_1 + z_2 = (x_1 \pm x_2) + i(y_1 + y_2)$

IV) $z_1 z_2 = (x_1 + iy_1)(x_2 \pm iy_2)$

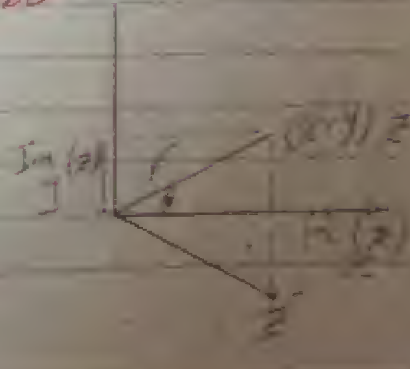
V) Divi $z_1 / z_2 = \dfrac{x_1 + iy_1}{x_2 + iy_2}$

VI) Conjugate Complex number

$z = x + iy \to \bar{z} = x - iy$

a) $\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$ b) $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$ D) $\overline{z_1 / z_2} = \bar{z}_1 / \bar{z}_2$ $z_2 \neq 0$

Geomettic interperation of Complex number



- the Polar form of Complex number →

1) the absolute value of z $[|z|]$

$$|z| = \sqrt{x^2 + y^2} = r$$

a) $|z| = 0 \rightarrow z = 0$ b) $|z_1 z_2| = |z_1||z_2|$

c) $|z_1 / z_2| = |z_1| / |z_2|$ d) $|z_1 + z_2| \leq |z_1| + |z_2|$

e) $|z_1 - z_2| \geq ||z_1| - |z_2||$

the Polar form $z = x + iy$

$$z = |z| e^{i\theta} = r e^{i\theta} \qquad r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1} y/x$$

$$z = r e^{i\theta} = r\text{"}\cos\theta + i\sin\theta\text{"}$$

= Dot Product ضرب قياسي =

$$z_1 \odot z_2 = |z_1||z_2| \cos\theta = x_1 x_2 + y_1 y_2 = R_e\text{"}\bar{z}_1 z_2\text{"}$$
$$= \tfrac{1}{2}(\bar{z}_1 z_2 + z_1 \bar{z}_2)$$

= Cross Product ضرب اتجاهي =

$$z_1 \times z_2 = |z_1||z_2| \sin\theta = x_1 y_2 - x_2 y_1 = Im(\bar{z}_1 z_2)$$
$$= \tfrac{1}{2}[\bar{z}_1 z_2 - \bar{z}_2 z_1]$$

★ Orthogonality ⟶ $z_1 \odot z_2 = 0$ $z_1 \perp z_2$

★ Parallel ⟶ $z_1 \times z_2 = 0$ $z_1 \parallel z_2$

★ Area of Parallelogram $A = |z_1 \times z_2|$

= Demoivre theorem =

i] $(\cos\theta \pm i\sin\theta)^n = \cos n\theta \pm i\sin n\theta$

II) $(\cos\theta \pm i\sin\theta)^{-n} = \cos(-n)\theta + i\sin(-n)\theta$

the Root of the Complex Number

$w^n = z$ $w = u + iv$ $z = x + iy$

- $z = x+iy = re^{j\theta} = r(\cos\theta + i\sin\theta)^{n}$
- $z^{1/n} = r^{1/n}(\cos\theta + i\sin\theta)^{1/n}$ $\quad w = r^{1/n}\left[\cos\frac{\theta+2\pi k}{n} + i\sin\frac{\theta+2\pi k}{n}\right]$

  $: (k = 0, 1, 2, 3, \cdots)$

$w = z^{1/2}$